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Leptogenesis and a Jarlskog invariant

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Abstract: The relation between low energy CP violating phases, and the CP asymmetry of leptogenesis ϵ_1, is investigated. Although it is known that in general those are independent, there may be a relation when a model is specified. We construct a Jarlskog invariant which is proportional to ϵ_1 if the right-handed neutrino masses are hierarchical. Since the invariant can be expressed in terms of left-handed neutrino parameters — some measurable, and some not — it is useful in identifying the limits in which ϵ_1 is related to MNS phases.

Keywords: Baryogenesis, Neutrino Physics, CP violation.

Contents

1. Introduction

There has been recently some interest in relating the baryon asymmetry produced via leptogenesis to leptonic observables. This connection is not straightforward, because leptogenesis [1] depends on the masses and couplings of the heavy right-handed (RH) neutrinos, whereas we can measure parameters of the light left-handed (LH) lepton doublets. Specifically, in thermal leptogenesis [1]–[3], with hierarchical ν_Rs, one needs the mass and eigenvector of the lightest ν_R to compute the baryon asymmetry.

CP violating phases in the leptonic sector could be observed in the coming years at a neutrino Factory, and appear in the neutrinoless double β decay matrix element.[1] The relation between observable phases and ϵ_1 has been explored in many papers, from various perspectives. There is a close connection in models with 2 right-handed neutrinos [5]. Using invariants, it was shown that there is no "direct" relation for three ν_R [6], in the sense that leptogenesis can work when there is no observable leptonic CP violation at low energy, and vice versa. For three hierarchical ν_Rs, and hierarchical Dirac Yukawa coupling constants, there is an analytic approximation that relates the ν_R and ν_L sectors [7, 8]. This was used [7] to relate low-energy phases to leptogenesis in SO(10) models, and used in a bottom-up phenomenological discussion for SUSY with universal soft masses [8]. Many analyses have been performed in left-right models [9], various grand unified theories [10] and/or textures [11] or particular models of the RH mass matrix [12].[2]

The $C\!\!\!/P$ of leptogenesis is among the ν_R, so is not easily related to the phases of the left-handed sector. This paper attempts to circumvent this problem by using a Jarlskog invariant (closely related to those introduced by Branco Lavoura and Rebelo [17]). For

[1] See [4] for a discussion of the (im)possibility of observing this phase.

[2] Reconstructing the heavy sector in SUSY [13] and non-SUSY [14, 15] seesaws, and in other ν_L mass generation scenarios [16] has also been discussed, with various assumptions for the unobserved LH parameters.

hierarchical ν_R, the CP asymmetry ϵ_1 produced in ν_{R1} decay, is the invariant multiplied by a factor depending on the ν_{R1} mass and decay rate. The invariant is a trace, so summed on all LH and RH indices. It can therefore be evaluated in terms of LH or RH parameters. The expression in terms of LH parameters is tractable, and identifies which phases of the left-handed sector contribute to ϵ_1. The magnitude of the invariant depends on unknown eigenvalues of the neutrino Yukawa matrix.

Invariants are interesting, in theories with many complex couplings, because they better encode the CP violation of physical processes than do parametrization-dependent phases. They also avoid potential confusion arising from basis and phases choices. Since our invariant is summed on all indices, it can be evaluated in any parametrization of the seesaw, with any basis choice and phase convention. Evaluated using its left-handed indices, the invariant depends on the phases of a matrix $W = V_L U$, where U is the MNS matrix, and V_L is unknown. It is therefore useful for seeing in which models (which V_L), the MNS phases are important for leptogenesis. In the "best case", the leptogenesis invariant is proportional to sin(a neutrinoless double beta decay phase). The relations we find agree with most previous results. What is new, is that by using the Jarlskog invariant, we can write the "$C\!\!\!/P\!\!\!/$ of leptogenesis" as a sum of terms involving LH phases. This is model-independent, and only requires that the ν_R be hierarchical. The expression for ϵ_1 in terms of the invariant may also have other interesting uses, discussed in the second part of section 4. For instance, the upper bound on ϵ_1 for degenerate ν_L, in the (not maximally restrictive) version of [18], can be trivially obtained.

The next section 2 contains notation and more background. The Branco-Lavoura-Rebelo (BLR) invariant, its relation to ϵ_1 and various formulae for the "leptogenesis invariant" are in section 3. In section 4 we consider what this invariant could be good for, and areas of parameter space where it can be written in terms of leptonic observables. Section 5 discusses and summarizes the result.

2. Background and notation

The lagrangian for the leptonic sector, in a seesaw [19] model with three right-handed neutrinos, can be written

$$\mathcal{L} = Y_e \bar{e}_R H_d \cdot \ell_L + Y_\nu \bar{\nu}_R H_u \cdot \ell_L + \frac{M}{2}\nu_R \nu_R + \text{h.c.}\,, \tag{2.1}$$

where the index order on the Yukawa matrices is right-left. We assume there are no SU(2) triplet scalars, whose vevs could give Majorana masses to the ν_L directly.

Two relevant bases for the ν_R vector space are the one where the mass matrix M is diagonal ($= D_M$), and where the Yukawa matrix $Y_\nu Y_\nu^\dagger$ is diagonal ($= D_Y^2$). The unitary matrix V_R transforms between these bases, so in the mass eigenstate basis

$$Y_\nu Y_\nu^\dagger = V_R^\dagger D_Y^2 V_R\,. \tag{2.2}$$

At low energies, well below the ν_R mass scale, the light (LH) neutrinos acquire an effective Majorana mass matrix $[m_\nu]$. In the vector space of LH leptons, there are three interesting bases — the one where the charged lepton Yukawa $Y_e^\dagger Y_e$ is diagonal, the one where the neutrino Yukawa $Y_\nu^\dagger Y_\nu$ is diagonal, and the basis where $[m_\nu]$ is diagonal. The first (D_{Y_e}) and last (D_m) are phenomenologically important, and are related by the MNS matrix U: $[m_\nu] = U^* D_m U^\dagger$ in the D_{Y_e} basis. The second (D_{Y_ν}) relative to the first (D_{Y_e}) can be important for phenomenology in SUSY models, where $Y_\nu^\dagger Y_\nu (\equiv V_L^\dagger D_{Y_\nu}^2 V_L$ in the D_{Y_e} basis) induces flavour violation via its appearance in the slepton RGEs [20]. The second and third are useful to relate LH and RH seesaw parameters[21]; the matrix W transforms between these bases. So let us concentrate on these last two bases for calculating invariants in this and the next section 3, and introduce the charged lepton mass eigenstate basis in the discussion, when relating leptogenesis to low energy leptonic CP violation.

In the basis where Y_ν is diagonal, $[m_\nu]$ can be written

$$\kappa \, \langle H_u \rangle^2 \equiv [m_\nu] = D_Y M^{-1} D_Y \langle H_u \rangle^2 = W^* D_\kappa W^\dagger \langle H_u \rangle^2 \, . \tag{2.3}$$

The matrix κ is convenient, to avoid Higgs vevs $\langle H_u \rangle$ in formulae. In this paper, κ will be in the D_{Y_ν} basis, unless otherwise stated.

Twenty-one parameters are required to fully determine the lagrangian of eq. (2.1). If Y_e is neglected, only 9 real numbers and 3 phases are required. These can be chosen in various ways:

1. "top-down" — input the ν_R sector: D_M, $D_{Y_\nu Y_\nu^\dagger}$, and V_R.

2. "bottom-up" — input the ν_L sector: D_κ, $D_{Y_\nu^\dagger Y_\nu}$, and W.

3. "intermediate" — the Casas-Ibarra parametrization [22]: D_M, D_κ, and a complex orthogonal matrix $R = D_M^{-1/2} Y_\nu D_\kappa^{-1/2}$.

To relate the RH parameters relevant for leptogenesis to the LH ones, many of which are accessible at low energy, it is useful to consider the first and second parametrization.

The baryon asymmetry of the Universe can be generated in the seesaw model, if enough ν_Rs decay out of equilibrium, and the CP asymmetry in the decay is large enough [1]. Thermal leptogenesis [2, 3], when the lightest right-handed neutrino ν_{R1} is produced by scattering interactions in the plasma, is an attractive and comparatively cosmology-independent way for this to occur. The final baryon asymmetry depends largely on three parameters: the ν_{R1} mass M_1, its decay rate Γ_1, and the CP asymmetry ϵ_1 in the decay. The decay rate Γ_j of ν_{Rj} can be conveniently parametrized as

$$\Gamma_j = \frac{[Y_\nu Y_\nu^\dagger]_{jj} M_j}{8\pi} \equiv \frac{\tilde{\kappa}_j M_j^2}{8\pi} \, , \tag{2.4}$$

where $\tilde{\kappa}_j$ is often of order the elements of κ (the ν_L mass matrix), although it is a rescaled

ν_R decay rate. The CP violating asymmetry is

$$\begin{aligned}\epsilon_1 &= \frac{\Gamma_1 - \bar{\Gamma}_1}{\Gamma_1 + \bar{\Gamma}_1} \\ &= -\frac{1}{8\pi[YY^\dagger]_{11}} \sum_j \mathrm{Im}\{[YY^\dagger]^2_{1j}\} g\left(\frac{M_j^2}{M_1^2}\right),\end{aligned} \tag{2.5}$$

where $\bar{\Gamma}_1$ is the decay rate into CP conjugate particles, and g is a kinematic function which in the MSSM at zero temperature (see [3] for more details) reads

$$g(x) = \sqrt{x}\left(\frac{2}{x-1} + \ln\left[1 + \frac{1}{x}\right]\right) \to \frac{3}{\sqrt{x}} + \frac{3}{2x^{3/2}} + \cdots . \tag{2.6}$$

The last limit occurs for $(M_1/M_j)^2 \ll 1$, $j = 2, 3$.

3. Results

A series of CP invariants for Majorana neutrinos [17], is

$$\mathrm{Im}\left\{\mathrm{Tr}\left[M^\dagger Y_\nu Y_\nu^\dagger (MM^\dagger)^m M Y_\nu^* Y_\nu^T\right]\right\} \tag{3.1}$$

for integers $m \geq 1$. These can be made more appropriate for leptogenesis with a slight modification. In the D_Y basis,

$$M = V_R D_M V_R^T\,, \qquad M^{-1} = V_R^* D_M^{-1} V_R^\dagger \tag{3.2}$$

so M can be replaced by M^{*-1} in the expression (3.1):

$$I_n = \mathrm{Im}\left\{\mathrm{Tr}\left[M^{-1} Y_\nu Y_\nu^\dagger M^{-1*} (M^{-1} M^{-1*})^n Y_\nu^* Y_\nu^T\right]\right\}. \tag{3.3}$$

It is clear that none of these is the CP asymmetry ϵ_1 of thermal leptogenesis, which for sufficiently hierarchical RH neutrinos, can be written

$$\epsilon_1 \simeq -\frac{3M_1}{8\pi[Y_\nu Y_\nu^\dagger]_{11}} \mathrm{Im}\left\{\left[Y_\nu Y_\nu^\dagger\right]^2_{1j}\right\} \frac{1}{M_j}. \tag{3.4}$$

In the remainder of the paper, when we refer to ϵ_1, we will mean this approximation, unless otherwise stated. (A significant hierarchy in the ν_R is required for this approximation to be reliable, as discussed in [29].) ϵ_1 is also an invariant, in that it can be evaluated in any basis — provided the external index "1" is the lightest RH neutrino ν_{R1}. So it differs from the BLR invariants (eq. (3.1)), in that one must know the ν_{R1} mass and eigenvector to evaluate it. This complicates attempts to relate leptogenesis to low-energy CP violation. The invariants of eq. (3.3) do not suffer this problem; since all indices are summed in the traces, the invariants can be evaluated in the LH or RH vector spaces. Specifically, I_1 can be written in terms of LH quantities as

$$\begin{aligned}I_1 &= \mathrm{Im}\left\{\mathrm{Tr}\left[\kappa^\dagger \kappa \kappa^\dagger (Y_\nu^T Y_\nu^*)^{-1} \kappa (Y_\nu^\dagger Y_\nu)^{-1}\right]\right\} \\ &= \mathrm{Im}\left\{\mathrm{Tr}\left[W D_\kappa^3 W^T D_{Y_\nu}^{-2} W^* D_\kappa W^\dagger D_{Y_\nu}^{-2}\right]\right\}.\end{aligned} \tag{3.5}$$

There are similar invariants in [17], but with Y_e replacing Y_ν^{-1}. Using Y_ν^{-1} changes the matrix transforming from $D_\kappa \to D_Y$, and the relative weighting of terms in the sum.

In a two generation model, it is clear that $I_1 \propto \epsilon_1$. This is not surprising, because the two generation model only has one phase (if Y_e is neglected). This phase is also parametrized by the invariant (3.1), with $m = 1$, as discussed in [30]. The difference between I_1 and ϵ_1 in an arbitrary number of generations is that an RH index is fixed to "1" in ϵ_1, but summed in I_n, which therefore contains extra terms which are not proportional to ϵ_1. However, if the RH neutrino masses are hierarchical, we can ignore those extra terms because they are weighted by $(M_1/M_j)^{-(1+2n)}$. Explicitly for $n \to \infty$

$$\frac{M_1^{2+2n} I_n}{[YY^\dagger]_{11}} \to \frac{8\pi}{3}\epsilon_1 \,. \tag{3.6}$$

The invariant I_n can be written, in the ν_R mass eigenstate basis, as

$$\begin{aligned} \frac{M_1^{2n+2} I_n}{[Y_\nu Y_\nu^\dagger]_{11}} &= \frac{-1}{[Y_\nu Y_\nu^\dagger]_{11}} \left[\mathrm{Im}\{[YY^\dagger]_{12}^2\} \frac{M_1}{M_2}\left(1 - \frac{M_1^{2n}}{M_2^{2n}}\right) + \mathrm{Im}\{[YY^\dagger]_{13}^2\} \frac{M_1}{M_3}\left(1 - \frac{M_1^{2n}}{M_3^{2n}}\right)\right] + \\ &\quad + \frac{-1}{[Y_\nu Y_\nu^\dagger]_{11}} \left(\frac{M_1}{M_2}\right)^{2n+2} \left[\mathrm{Im}\{[YY^\dagger]_{23}^2\} \frac{M_2}{M_3}\left(1 - \frac{M_2^{2n}}{M_3^{2n}}\right)\right] && (3.7) \\ &\simeq \frac{8\pi}{3}\epsilon_1 + \text{extra}\,. && (3.8) \end{aligned}$$

The ν_R hierarchy must be steep enough, or n large enough, to ensure that the magnitude of the extra bits is much less than $8|\epsilon_1|$. The second line can be larger than the $O(M_1^2/M_j^2)$ terms appearing in the first line, and is bounded above:

$$\text{extra} \leq \left(\frac{M_1}{M_2}\right)^{2n} \frac{\tilde{\kappa}_2}{\tilde{\kappa}_1} M_1 \kappa_3 \tag{3.9}$$

(where $\tilde{\kappa}_j$ is defined in eq. (2.4)). If $\tilde{\kappa}_1$ is small, and $\tilde{\kappa}_2$ is large, then $(M_1/M_2)^{2n}$ must be small. Writing $\tilde{\kappa}_2 = |R_{2j}|^2 \kappa_j$, we see that $\tilde{\kappa}_2/\tilde{\kappa}_1$ is large for large imaginary θ_{23} in R, which is bounded above by requiring that $|Y_{33}| = |R_{33}\sqrt{M_3\kappa_3}| \leq 1$. This implies $\tilde{\kappa}_2 \lesssim 1/M_3$. So for $\tilde{\kappa}_1 \gtrsim 10^{-2}\kappa_3$, and $\epsilon_1 \gtrsim 10^{-6}$ we obtain that I_1 will give a reasonable approximation for ϵ_1 if $(M_1/M_2)^3 < 10^{-7}$. That is,

$$\epsilon_1 \simeq \frac{3M_1^3}{8\pi\tilde{\kappa}_1} I_1 \qquad \text{for} \quad 200 M_1 < M_2\,. \tag{3.10}$$

See the appendix for a more detailed discussion. We here draw attention to two points. First, there are $O(M_1^2/M_j^2)$ corrections to the ϵ_1 of eq. (3.4), similar to those in parentheses in line (3.7), arising from the expansion of g in eq. (2.5); see eq. (2.6). These corrections are always present, and small if eq. (3.10) is satisfied. So if we used the invariant I_2, the second line (eq. (3.7)) would be negligible for a less steep hierarchy, but we would have to check that the M_1^2/M_2^2 terms were small enough. So it is reasonable to work with the simplest invariant, I_1. Second, in the parameter space where $\tilde{\kappa}_2 \gg \kappa_3$, the "physical" ν_R parameters (masses M_i and Yukawa couplings Y_{ij}) are tuned against each other (recall that in the ν_R and ν_L mass eigenstate bases, $\tilde{\kappa}_2 = [YY^\dagger]_{22}/M_2$ and $\kappa_2 = [Y^T M^{-1} Y]_{22}$). These cancellations appear less unnatural when $M_2 \sim M_3$ (see the model in the appendix), so we did not impose $M_2 \ll M_3$ in deriving the bound (3.10). On the other hand, if we reject

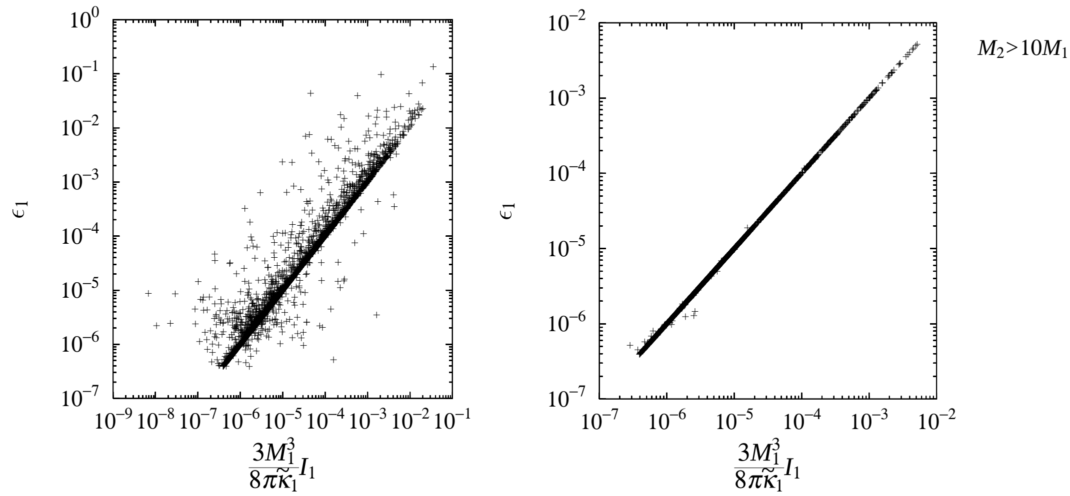

Figure 1: The scatter plot of ϵ_1 and I_1. We randomly generate the parameters which reproduce correct neutrino masses, mixing angles and large enough value of the baryon asymmetry of the universe. In the right figure, we further require the mild hierarchy of $M_2 > 10M_1$.

such tuning, then $\tilde{\kappa}_2 \sim \kappa_3$ in eq. (3.9), and I_1 is a good approximation for $30M_1 < M_2$. We demonstrate this by showing plots in figure 1, where we randomly generate Y_ν and M_i which reproduce correct neutrino masses, mixing angles and large enough ϵ_1. In the left plot, the correlation between ϵ_1 and I_1 is weak due to the extra factor in eq. (3.8), whereas nearly perfect linear relation can be seen in the right plot where we imposed a mild hierarchy $10M_1 < M_2$.

In principle, it is interesting to notice that ϵ_1 can be expressed only in terms of LH parameters: if the ν_R masses satisfy eq. (3.9), then

$$\begin{aligned}\frac{\left[YY^\dagger\right]_{11}}{M_1^4} &\simeq \sum_j \frac{\left[YY^\dagger\right]_{jj}}{M_j^4} = \mathrm{Tr}\left[M^{*-1}M^{-1}M^{*-1}M^{-1}YY^\dagger\right] \\ &= \mathrm{Tr}\left[\kappa^\dagger\kappa(Y^TY^*)^{-1}\kappa^\dagger(Y^\dagger Y)^{-1}\kappa(Y^TY^*)^{-1}\right] \equiv T\,. \end{aligned} \tag{3.11}$$

So $\epsilon_1 \simeq 3I_1/(8\pi T)$. In practise, T is a complicated function of unmeasured parameters. It looks recognisable in some limiting cases, such as $V_L = 1$ with negligeable y_1^2/y_j^2, where $T \simeq (|U_{ej}|^2 m_j^2)|m_{ee}|^2/y_1^6$.

To summarize: we can write ϵ_1 proportional to a Jarlskog invariant. The coefficient can be written as a function of the mass and decay rate of the ν_{R1}, both of which are required elsewhere in thermal leptogenesis calculations, or as a (more complicated) function of parameters from the LH sector, such as eigenvalues of Y_ν. The invariant can be written in terms of LH parameters, so might tell us about the relative importance of different LH phases for leptogenesis.

Let's try to evaluate I_1 on its LH indices. This invariant vanishes for degenerate masses or Yukawa eigenvalues (although CP violation is possible among degenerate Majorana neutrinos [23]). So D_Y^{-2} in (3.5) can be written

$$D_Y^{-2} = \frac{1}{y_3^2} I + \begin{bmatrix} 1/y_1^2 - 1/y_3^2 & 0 & 0 \\ 0 & 1/y_2^2 - 1/y_3^2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3.12}$$

and it is easy to see that in evaluating I_1, D_Y^{-2} can be replaced by the second matrix on the RHS. This simplifies the formula because it removes a row:

$$\begin{aligned} I_1 = \kappa_3\kappa_2 \left[\kappa_3^2 - \kappa_2^2\right] \mathrm{Im} \left\{ \left(\frac{(y_3^2 - y_1^2)}{y_1^2 y_3^2} W_{13} W_{12}^* + \frac{(y_3^2 - y_2^2)}{y_2^2 y_3^2} W_{23} W_{22}^* \right)^2 \right\} + \\ + \kappa_3\kappa_1 [\kappa_3^2 - \kappa_1^2] \mathrm{Im} \left\{ \left(\frac{(y_3^2 - y_1^2)}{y_1^2 y_3^2} W_{13} W_{11}^* + \frac{(y_3^2 - y_2^2)}{y_2^2 y_3^2} W_{23} W_{21}^* \right)^2 \right\} + \\ + \kappa_2\kappa_1 [\kappa_2^2 - \kappa_1^2] \mathrm{Im} \left\{ \left(\frac{(y_3^2 - y_1^2)}{y_1^2 y_3^2} W_{12} W_{11}^* + \frac{(y_3^2 - y_2^2)}{y_2^2 y_3^2} W_{22} W_{21}^* \right)^2 \right\}, \end{aligned} \tag{3.13}$$

where W is defined in eq. (4.1), and κ in eq. (2.3). This is the main result. In the next section 4, we will discuss how to simplify it.[3] The formula for I_1 is more compact if we replace $(y_j^{-2} - y_3^{-2}) \to y_j^{-2}$, for $j = 1, 2$. This amounts to dropping corrections of order $(y_1/y_3)^2$ and $(y_2/y_3)^2$ to terms contributing to ϵ_1. If the eigenvalues of Y_ν are hierarchical, this is an insignificant modification.

It is also possible to express I_n in terms of the matrix R:

$$I_n = -\,\mathrm{Im}\left\{ \mathrm{Tr}\left[D_\kappa^2 R^T D_M^{-2n} R \right] \right\}, \tag{3.14}$$

where D_κ^2 can be replaced by $D_{\Delta\kappa^2}$, similarly to (3.12). Taking $n \to \infty$, eq. (3.14) gives an expression for ϵ_1 in terms of R

$$\epsilon_1 = -\frac{3M_1}{8\pi\tilde{\kappa}_1} \mathrm{Im}\left\{ R_{1j}^2 (\kappa_j^2 - \kappa_3^2) \right\} \tag{3.15}$$

which looks slightly different from the formula in [18].

4. Relating ϵ_1 to MNS phases

Figure 1 tells us that (3.13) is an almost exact formula for $(8\pi\tilde{\kappa}_1/3M_1^3) \times \epsilon_1$. However, the magnitude of ϵ_1 depends on unknown parameters, and eq. (3.13) for the invariant is not illuminating. In this section, we first assume that ϵ_1 is large enough, and study what I_1 can tell us about low energy CP violation in various cases. Then we will consider uses of the alternate expression (3.15) for ϵ_1.

The CP violation required for leptogenesis is encoded in I_1. The expression in terms of LH parameters is simple, but of these parameters, only two mass differences $\kappa_i^2 - \kappa_j^2$ are measured. We need to identify a few important terms in the sum, and express them in terms of potentially observable phases, if we wish to relate the CP violation required for leptogenesis to phases in the MNS matrix.

[3] The approximation for ϵ_1 used in [13] can be obtained from the first,third and fifth terms of eq. (3.13).

The matrix W transforms from the LH neutrino mass eigenstate basis to to the basis where $Y_\nu^\dagger Y_\nu$ is diagonal. It can be written as

$$W = V_L U \,, \tag{4.1}$$

where U is the MNS matrix, parametrized as

$$U = \hat{U} \cdot \mathrm{diag}(1, e^{i\alpha}, e^{i\beta}) \,. \tag{4.2}$$

α and β are "Majorana" phases, and $\hat{U}$ has the form of the CKM matrix

$$\hat{U} = \begin{bmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13} \end{bmatrix} . \tag{4.3}$$

W is parametrized in the same form as U.

The matrix V_L diagonalizes $Y_\nu^\dagger Y_\nu$ in the charged lepton mass eigenstate basis

$$V_L Y_\nu^\dagger Y_\nu V_L^\dagger = D_Y^2 \,. \tag{4.4}$$

In SUSY models, some information may be available about the $[V_L]_{31}$ and $[V_L]_{32}$ elements from slepton-mediated lepton flavour violation [20, 22, 24].

The phase δ induces a CP-violating asymmetry in neutrino oscillation rates. For sufficiently large δ and θ_{13}, this could be observed at neutrino beams currently under consideration [25]. The "Majorana" phases α and β can only affect lepton number violating processes [26], which have yet to be observed. Both these phases appear in the neutrinoless double beta decay amplitude, which is proportional (when squared) to

$$\begin{aligned} |m_{ee}|^2 &= |U_{ej}^2 m_j|^2 \\ &= c_{13}^4 c_{12}^4 m_1^2 + c_{13}^4 s_{12}^4 m_2^2 + s_{13}^4 m_3^2 + 2m_1 m_2 c_{13}^4 c_{12}^2 s_{12}^2 \cos 2\alpha + \\ &\quad + 2m_2 m_3 s_{13}^2 c_{13}^2 s_{12}^2 \cos 2(\beta - \delta - \alpha) + 2m_1 m_3 s_{13}^2 c_{13}^2 c_{12}^2 \cos 2(\beta - \delta) \,, \end{aligned} \tag{4.5}$$

where m_{1-3} are the neutrino masses. The $0\nu 2\beta$ decays are CP conserving, so the cosine of the phases appears in the cross terms. Detecting these cross terms, and extracting some information about the "Majorana" phases, does not appear possible [4].

If there is one dominant term in the invariant, then I_1 is proportional to $\sin(2\xi)$ where ξ is a combination of the phases of the unitary matrix W. The combination ξ, in various limits, is listed in table 1. The cases are labeled by the ν_L mass pattern and a choice of the form of V_L, and then constructed by looking for conditions such that one of the six terms in eq. (3.13) dominates I_1. In the limits where two terms have the same order of magnitude, we do not identify a "phase". In the table, and all the following, we assume that phases in V_L and U are $O(1)$. The table tells us that the leptogenesis invariant can be controlled by MNS phases if the off-diagonal elements of V_L are "small". In this case, $W_{ij} \sim U_{ij}$, except for W_{13}, where elements of V_L could make the leading contribution. There are various interesting limits.

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ν_L masses	$V_L \simeq$	1	2	3	4	5	6
hierarchical $m_3 \gg m_2 > m_1 = m_s$	I	W_{13}^2 $\delta_{(W)} - \beta + \alpha$	y_1^4/y_2^4 $\alpha - \beta$			$\Delta_{\rm sol}^2 m_s/\Delta_{\rm atm}^3$ $-\alpha$	
	U^{-1}	$W_{12}^2 W_{13}^2$ $\delta_W - \beta_W + \alpha_W$	$y_1^4 W_{23}^2/y_2^4$ $\alpha_W - \beta_W$	$W_{13}^2 m_s/\Delta_{\rm sol}$ $\delta_W - \beta_W$		$W_{12}^2 \Delta_{\rm sol}^2 m_s/\Delta_{\rm atm}^3$ $-\alpha_W$	
inverse hier. $m_2 > m_1 \gg m_3 = m_s$	I	$m_s W_{13}^2$ $-$	$m_s y_1^4/y_2^4$ $-$	$m_s W_{13}^2$ $-$	$m_s y_1^4/y_2^4$ $-$	$\frac{\Delta_{\rm sol}^2}{\Delta_{\rm atm}}$ $-\alpha$	
	U^{-1}		$m_s W_{23}^2 y_1^4/y_2^4$ $\alpha_W - \beta_W$	$m_s W_{13}^2$ $\delta_W - \beta_W$		$W_{12}^2 \frac{\Delta_{\rm sol}^2}{\Delta_{\rm atm}}$ $-\alpha_W$	
degenerate $m_3 \gtrsim m_2 \gtrsim m_1$	I	W_{13}^2 $-$	y_1^4/y_2^4 $-$	W_{13}^2 $-$	y_1^4/y_2^4 $-$	$\frac{\Delta_{\rm sol}^2}{\Delta_{\rm atm}^2}$ $-\alpha$	
	U^{-1}		$W_{23}^2 y_1^4/y_2^4$ $\alpha_W - \beta_W$	W_{13}^2 $\delta_W - \beta_W$		$W_{12}^2 \frac{\Delta_{\rm sol}^2}{\Delta_{\rm atm}^2}$ $-\alpha_W$	

Table 1: The combination of phases which contribute to leptogenesis in several limiting cases. The numbers 1–6 represent the dominant terms in eq. (3.13). Reading along a row, the upper entries are the relative order of magnitude of the terms. If one term dominates, then $\epsilon_1 \propto \sin$(phase written below) + small corrections. (Δ_{sol}^2 is the solar mass squared difference, and the phase convention of W is that of eqns. (4.2) and (4.3)).

1. suppose that $V_L \sim 1$, so the 12 and 23 angles in W are large. The first, second and fifth terms in eq. (3.13) could be important, with relative sizes $\sim W_{13}^2 : (y_1/y_2)^4 : \frac{\kappa_1(\kappa_2^2-\kappa_1^2)}{\kappa_3(\kappa_3^2-\kappa_2^2)}$. The case where the first term dominates can be particularly interesting:

$$\epsilon_1 \simeq \frac{3}{8\pi\tilde{\kappa}_1} M_1^3 I_1 \simeq \frac{3M_1^3}{8\pi\tilde{\kappa}_1} \frac{\kappa_2\kappa_3}{y_1^4} [\kappa_3^2 - \kappa_2^2] c_{13}^2 s_{12}^2 |W_{13}|^2 \sin(2\beta - 2\delta_W - 2\alpha)\,, \tag{4.6}$$

   where δ_W is the phase of $\hat{W}_{13} \simeq \hat{U}_{13} + V_{L12}/\sqrt{2} + V_{L13}/\sqrt{2}$. If any one of the terms contributing to $\hat{W}_{13}$ dominates, then the phase of $\hat{W}_{13}$ is the phase of that term (with our preliminary assumption that all phases are large).

   (a) For instance, if $\theta_{13} \gg \theta_{L13}, \theta_{L12}$, then $\delta_W \simeq \delta$ is the Dirac phase of the MNS matrix, and $\delta - \beta + \alpha$ is the phase of the $m_2 m_3$ term in the $0\nu 2\beta$ decay matrix element.

   (b) If θ_{L12} or $\theta_{L13} \gg \theta_{13}$, then the phase δ_W is dominated by $\not{C}\not{P}$ from V_L, so δ is "subdominant" for leptogenesis (because it multiplies s_{13}), and $\alpha - \beta$ multiplies W_{12} so could make some "contribution" to the CP violation required for leptogenesis (see [8] for a more detailed discussion).

2. Another simplifying limit is to take $\kappa_1 \to 0$. If this occurs for fixed y_1 ($M_3 \to \infty$), the first two terms of eq. (3.13) remain. This is the case studied in [5].

3. Now consider the possibility that $V_L \sim U^{-1}$, so $\hat{W} = I + E$, where the matrix elements of E are small. This can arise in (SU(5) texture) models where the mixing in the lepton doublet sector is large, so the MNS matrix has large angles because $V_L^\dagger \simeq U$. These parameters are interesting for leptogenesis, because for fixed M_1, the

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baryon asymmetry is maximized for $\tilde{\kappa}_1 \gtrsim \kappa_2/10$ (The asymmetry produced in ν_{R1} decay peaks for W close to, but not equal to I. It rises as $W \to 1$, because less of it is washed out by inverse decay as $\tilde{\kappa}_1$ decreases, and also because $\epsilon_1 \propto 1/\tilde{\kappa}_1$ increases. However, in the hierarchical ν_R approximation of eq. (3.4), ϵ_1 drops rapidly to zero at $W = I$.). There is little relation between the phases of the MNS matrix and ϵ_1 in this case. This can easily be seen in the top-down approach: at the scale M_1, V_L has large angles, W has small angles, and the phases of W control leptogenesis. $U = V_L^\dagger W$, so the complex matrix elements of U will be sums of terms from V_L and W. In particular, the Majorana phases of U will be those of W, plus those of $V_L^\dagger \hat{W}$.

We now come to implications of eq. (3.15). In the hierarchical ν_R approximation, where $g(x)$ is taken as the first term of eq. (2.6), there is an upper bound [27, 18] on ϵ_1 :

$$\epsilon_1 \leq \frac{3}{8\pi} M_1 (\kappa_3 - \kappa_1)\,. \tag{4.7}$$

A stronger bound was presented in [28], but see [29] for a careful discussion. Using eq. (3.15), it is trivial to reproduce the degenerate ν_L ($m_3 > m_2 > m_1 \sim m$) bound of [18]:

$$|\epsilon_1| \leq \frac{3M_1}{8\pi\kappa_1} \sum_j \mathrm{Im}\{R_{1j}^2\}(\kappa_3^2 - \kappa_j^2) \leq \frac{3M_1}{16\pi\langle H_u\rangle^2} \frac{\Delta_{atm}^2}{m}\,, \tag{4.8}$$

where the first inequality follows from $\tilde{\kappa}_1 \geq \kappa_1$ [18]. R is a complex orthogonal matrix, so the sum is $\leq \mathrm{Im}\{R_{13}^2\}\Delta_{atm}^2/(m\langle H_u\rangle^2)$, and max $\mathrm{Im}\{R_{13}^2\} = 1/2$, which gives the second inequality.

5. Discussion

The CP asymmetry ϵ_1 is the imaginary part of a complex number, so it is not clear whether there is a "leptogenesis phase": to have a phase, ϵ_1 would need a uniquely defined real part. This problem was neatly addressed in [27], by defining an "effective" $C\!\!\!/P\!\!\!/$ parameter $\delta_{\mathrm{HMY}} = \epsilon_1/\epsilon_1^{\max}$, where ϵ_1^{max} is the upper bound on ϵ_1 of eq. (4.7). δ_{HMY} has desirable behaviour in two limits; it goes to zero when there is no $C\!\!\!/P\!\!\!/$, and to 1 for maximal CP violation. However, one must tune the magnitude of CP conserving parameters, as well as phases, to obtain $\delta_{\mathrm{HMY}} \to 1$, so this definition has not accomplished the separation between CP conserving and CP violating parameters, which speaking about phases implies. Another approach is to consider specific models, where ϵ_1 can be written as a function of real parameters $\times$ sin(phases).

In $C\!\!\!/P\!\!\!/$ theories which contain many phases, the physical observables are combinations of complex couplings, which can be related to Jarlskog invariants. The latter are perhaps more useful than thinking about couplings of fixed magnitude which have different phases in different parameterizations. So in this spirit, we look for an leptogenesis invariant, rather than a leptogenesis phase.

The first thing that must always be said, in discussing potential connections between MNS phases and leptogenesis, is that there is no linear relation: leptogenesis can work when there is no $\not{C}\not{P}$ in MNS, and measuring low energy leptonic phases does not imply that there is CP violation available for leptogenesis. This was clearly and elegantly shown in [6]. Any relation depends on the choice of "independent" phases. If ϵ_1 and δ are defined to be "independent", then it follows that they are unrelated.[4] Or phrased in terms of invariants: at least six independent invariants are needed to determine the phases of the seesaw. If I_1 and the Jarlskog invariant $J \propto \mathrm{Im}\,\mathrm{Tr}[Y_e^\dagger Y_e, m_\nu^\dagger m_\nu]^3$ are in this set, then they have been chosen to be independent. This particular choice is disingenuous, but illustrates the pitfalls of choosing a parametrization, calculating ϵ_1, and drawing conclusions about the relations between leptogenesis and low-energy CP violation. Nonetheless, a connection between ϵ_1 and δ is interesting to proponents of a neutrino Factory — so what can we say?

For hierarchical ν_Rs, the baryon asymmetry produced in thermal leptogenesis is proportional to a Jarlskog invariant I_1. The invariant encodes the CP violation required for leptogenesis, and *can be written as a function of ν_L parameters.* It depends on the phases of a matrix $W = V_L U$, where U is the MNS matrix, and V_L is unknown. So a model for V_L is required to establish any relation between between ϵ_1 and δ. The coefficient relating ϵ_1 to I_1 can be written as a function of the ν_{R1} mass and decay rate, or of Y_ν eigenvalues, ν_L masses and the matrix W.

In some areas of parameter space, one term dominates in the invariant I_1. When these areas correspond to models, where the relation of ϵ_1 to MNS phases has been discussed, we reproduce their results. For instance, in the "best case" scenario, the leptogenesis phase is $\delta - \beta + \alpha$, the phase of the $m_2 m_3$ term of the neutrinoless double beta decay matrix element. This would arise if the angles of V_L are smaller than θ_{13} — e.g. if θ_{13} was measured close to its current bound, and no LFV decays $\ell_j \to \ell_i \gamma$ were seen (although superpartners were discovered to be light), then we could be in this scenario. Unfortunately, it is the *cosine* of $\delta - \beta + \alpha$ which appears in neutrinoless double beta decay, so there would be no correlation with the sign [31] of the baryon asymmetry.

Also, in the limit where the smallest ν_L mass goes to zero, the invariant becomes simpler, losing many of its terms. Or if there are large angles in the unknown matrix V_L (the rotation matrix from the basis where Y_e is diagonal, to where Y_ν is diagonal), then no simple relation between the MNS phases and leptogenesis is expected: the invariant contains many terms to which many phases contribute, and there are possible cancellations.

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[4]This arises in the Casas-Ibarra seesaw parametrization.

A. How steep is the ν_R hierarchy?

If we write $Y_\nu = \sqrt{D_M} R \sqrt{D_\kappa}$, where R is a complex orthogonal matrix and $M_1 < M_2 < M_3$ and $\kappa_1 < \kappa_2 < \kappa_3$ are positive, then eq. (3.9) becomes

$$\left(\frac{M_1}{M_2}\right)^2 \ll \frac{\tilde{\kappa}_1}{\tilde{\kappa}_2} \frac{8\pi\epsilon_1}{3\kappa_3 M_1} = \frac{|R_{1j}|^2 \kappa_j}{|R_{2k}|^2 \kappa_k} \delta_{\rm HMY}\,, \tag{A.1}$$

where $\delta_{\rm HMY} \leq 1$, see eq. (4.7) with $\kappa_1 \ll \kappa_3$. We look for parameters that minimize the RHS, to determine how steep a hierarchy is required between M_1 and M_2. If $\tilde{\kappa}_1$ decreases, this makes the RHS smaller, but we impose $\tilde{\kappa_1} \gtrsim \kappa_3/100$ for two reasons: 1) the baryon asymmetry decreases for $\tilde{\kappa}_1$ much below this value, because not enough ν_{R1} are produced, and 2), some tuning is required to get $\tilde{\kappa}_1 < \kappa_2$, because the MNS angles are large. Instead, $\tilde{\kappa}_2$ can be increased, $|R_{2k}|^2 \kappa_k \sim e^{2\eta}\kappa_3$, when the "12" angle of R is real, the "13" angle zero, and the "23" angle acquires a large imaginary part $\theta_{23} = \rho + i\eta, \eta \gg 1$. Imposing that the magnitude of $[Y_\nu]_{33}$ should be ≤ 1, gives an upper bound on η :

$$e^\eta \lesssim \frac{1}{\sqrt{\kappa_3 M_3}}$$

so $\tilde{\kappa}_2 \leq 1/M_3$. Thermal leptogenesis requires $\epsilon_1 \gtrsim 10^{-6}$ [2, 3], so eq. (3.10) will be a good approximation, for $n = 1$, if

$$\left(\frac{M_1}{M_2}\right)^3 \frac{M_2}{M_3} < 10^{-7}\,.$$

We allow $M_2 \sim M_3$, with $y_3 \gg y_2$, because this seems a natural way to obtain the cancellations implied by $\tilde{\kappa}_2 \gg \kappa_3$. If

$$\mathcal{M} = \begin{bmatrix} 0 & M \\ M & 0 \end{bmatrix}, \qquad Y = \begin{bmatrix} y_2 & 0 \\ 0 & y_3 \end{bmatrix},$$

then $\kappa_2 = \kappa_3 = y_2 y_3/M$ and $\tilde{\kappa}_2 = (y_2^2 + y_3^2)/2M$. (Notice, however, that line (3.7) is $\propto M_2^2 - M_3^2$, so is zero for $M_2 = M_3$.)

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